IMPROVEMENT OF LEMAITRE'S CREEP LAW TO ASSESS THE SALT MECHANICAL BEHAVIOR FOR A LARGE RANGE OF THE DEVIATORIC STRESS

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Abstract

This paper describes the investigations made to reproduce with the strain-hardening transient Lemaitre's law the creep of a rock salt tested in laboratory under different levels of the deviatoric stress. The results obtained showed that, when fitted on a given range of the mechanical loading, the law underestimates the deformational response of the sample for the opposite range of loading. An improvement was brought to the Lemaitre's law to integrate the contribution of both low and high deviatoric stresses. This new development was implemented into a numerical software in order to simulate the impact of deviatoric stresses on the thermo-mechanical behavior of salt caverns.

1. Introduction

The strain-hardening transient Lemaitre's creep law (also called Menzel-Schreiner law) is commonly used to describe the viscoplastic behavior of salt rock submitted to mechanical and thermal loadings. The complete set of parameters of the law are usually determined through a triaxial creep test with different stages of deviatoric stress and temperature by fitting the measured strain variation with time.

The deviatoric stresses commonly applied in laboratory are ranging between 5 and 20 MPa. The salt behavior for low deviatoric stresses 0-5 MPa, which correspond to the range of primary interest for assessing cavern behavior, is extrapolated from the parameters deduced from the laboratory testing (Berest et al, 2008).

A testing program has been conducted in the laboratory of Ecole des Mines de Paris on a rock salt prone to high creep to check its deformational response under different levels of the deviatoric stress. The first attempt to fit the standard Lemaitre's creep law on the data was not satisfying since the constitutive law was not able to reproduce the complete strain versus time curve. In particular, it has been found that the Lemaitre's law underestimates the creep strain for low deviatoric stresses. When fitted on low deviatoric stresses, the Lemaitre's law underestimated once again the deformational response of the sample under the higher range of mechanical loading. Although not investigated in detail in this paper, this observation is the result of the activation of well-known different mechanisms of creep deformation at different stress levels (Hunsche 1984, Munson and Dawson 1984, Langer 1984 and Blum and Fleischman 1988).

As a result, an improvement was brought to the Lemaitre's law to integrate the contribution of both low and high deviatoric stresses. This modification allowed to reproduce properly the laboratory tests conducted for a large range of the deviatoric stress. This new development was also implemented into the GEO1D software of Ecole des Mines de Paris, which is dedicated to simulate the thermo-mechanical behavior of salt caverns for brine production, liquid hydrocarbons or natural gas storage, in order to analyze the impact of deviatoric stresses on cavern behavior.

2. Lemaitre's law and testing procedure

The time dependent deformation of the salt is analyzed by the strain hardening rheological model of Lemaitre which is characterized by the following features :

- the material is isotropic and elasto-viscoplastic ; it undergoes irreversible deformations when subjected to a deviatoric stress,
- the viscoplastic deformations occur without change of volume and the rate of the non-elastic strain tensor is parallel to the deviatoric stress tensor,
- a cylindrical sample, submitted to a constant deviatoric stress σ (difference between the axial pressure and the confining pressure) and a constant temperature T, undergoes a relative height reduction ε (axial strain) which takes the following expression :

$$\varepsilon = \frac{\sigma - \sigma_0}{E} + \varphi(\sigma)\,\psi(T)\,t^{\alpha} + \alpha_1(T - T_0) \qquad (1)$$

The elastic part of the total strain is related to the variation of the stress by the standard linear elastic law defined by the Young modulus. The thermal part is a linear function of the temperature characterized by the thermal dilation coefficient α_1. The creep part is due to the viscosity of the rock and obeys to a power law with time (exponent α). It is characterized by :

- $\psi(T)$ is an increasing function describing the effect of temperature on creep ; it takes in general the following form :

$$\psi(T) = \exp\left(-\frac{Q}{R}\left[\frac{1}{T} - \frac{1}{T_r}\right]\right) \text{ where :} \qquad (2)$$

Q is the energy of activation and the ratio Q/R has the dimension of a temperature and is expressed in Kelvin.

Various authors have shown the need to involve several mechanisms, i.e. several activation energies ($\psi. = \psi_1 + \psi_2 + ...$, Tijani 1987) to better reflect the influence of temperature.

- the viscoplastic strain is an increasing function of the deviatoric stress σ, and for the function $\varphi(\sigma)$, the power law proposed by Lemaitre is adopted

$$\varphi(\sigma) = \left(\frac{\sigma}{K}\right)^{\beta} \text{ : K and the } \beta \text{ are the parameters of the law} \qquad (3)$$

T_r is a reference temperature at which the parameter K is defined.

The multistage creep test is performed on a creep testing equipment including a triaxial cell made of steel, axial and radial hydraulic systems and a heating system. The sample, having a slenderness ratio (height to diameter ratio) of 2, is installed inside the cell and submitted to axial and radial stresses, applied by pressurized oil, and to a temperature. The test consists of several time intervals of conventional creep test performed on the same sample at constant stress and temperature. Each interval is separated by step-wise changes either in axial stress or temperature. The global axial strain is measured by L.V.D.T. (Fig. 1).

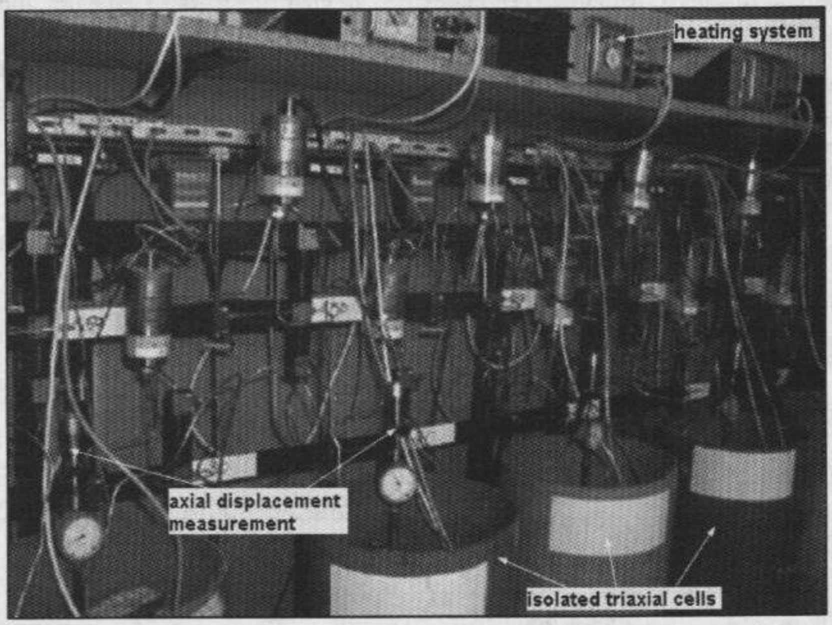

Figure 1 : Equipment of the multistage triaxial creep tests

In order to separate the phenomenon of thermal dilation from the effect of temperature on the viscoplastic behavior of the salt material, the test is carried out in long duration steps without changing at the same time the stresses and the temperature. Hence, the analysis of the instantaneous responses due to change of stress and temperature allows to deduce the elastic modulus and the thermal dilation coefficient respectively.

3. Multistage creep tests under different deviatoric stresses

The geological description of the core samples and the preliminary data indicated that the tested salt is marked by a high content of insoluble and is prone to high creep. The equipment was therefore adapted to be able to measure large deformations.

Before conducting the multistage triaxial creep tests, the following physical and mechanical properties of the salt were measured in laboratory (Tab. 1).

Table 1 : Average properties of the tested salt

Insoluble content (%)	Density (kg/m^3)	Sound velocity (m/s)	Tensile strength (MPa)	Young Modulus (MPa)	Poisson ratio
30	2255	4630	2.35	25000	0.3

Two main series of tests were carried out. The first one included two identical tests conducted under three steps of classical high deviatoric stresses (5, 10 and 15 MPa) at a constant temperature of 50 °C with a confining pressure of 10 MPa. The second series was focusing on the effect of low deviatoric stresses and corresponded to a first test with four steps of low deviatoric stresses (0.2, 0.5, 1 and 2 MPa) followed by a last step with a higher stress (10 MPa) ; the temperature was the same (50 °C) and the confining pressure was equal to 5 MPa. An additional test was conducted within this series to check the homogeneity of the salt.

The parameters of Lemaitre's law (α, β, K) were determined by a software which uses the method of the conjugate gradient to fit the data by minimization of the difference between the experiment and the theory within the principle of least squares (Hamami, 1993).

Table 2 summarizes the results of the two first samples and gives the corresponding creep index. This parameter is defined as the axial creep strain obtained after 1 year under a deviatoric stress of 10 MPa and is commonly used to compare the creep behavior of different salts (Tijani 2008). The results of the two samples are quite homogenous and are illustrated by figure 2.

Table 2 : Lemaitre's parameters and corresponding creep indexes

Sample	α	β	K (MPa at 50° C)	Creep index (%)
S1	0.502	4.343	1.559	7.3
S2	0.449	3.514	0.891	6.9

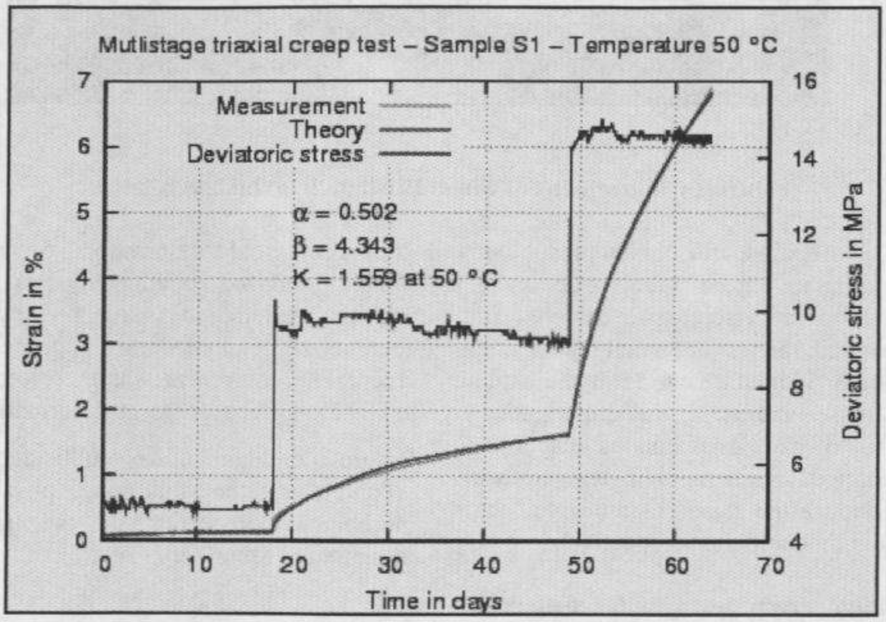

Figure 2 : Experimental and theoretical creep responses of sample S1

The comparison between the experimental and the theoretical strains shows that the parameters of Lemaitre's law derived from the whole test underestimate the creep at low values of the deviatoric stress. Furthermore, the creep indices, determined on the basis of these parameters, highlight that the salt is a high creeping material.

In order to investigate in details this effect, a third test was conducted with a more pronounced difference between the deviatoric stress steps. Figure 3 gives the same comparison when fitting the parameters of Lemaitre's law on all the steps of the deviatoric stress. This figure shows again that the law does not describe properly the salt behavior for the low stress range.

A second fitting was therefore performed on the data, but without accounting for the last step of the deviatoric stress (10 MPa). The results are illustrated by figure 4 and the corresponding parameters are given on table 3 in comparison with those computed from all the data. Unlike the previous result, the law reproduces properly the deformational response at low stresses, but a divergence is observed for the last step where data were not considered in the fitting operation.

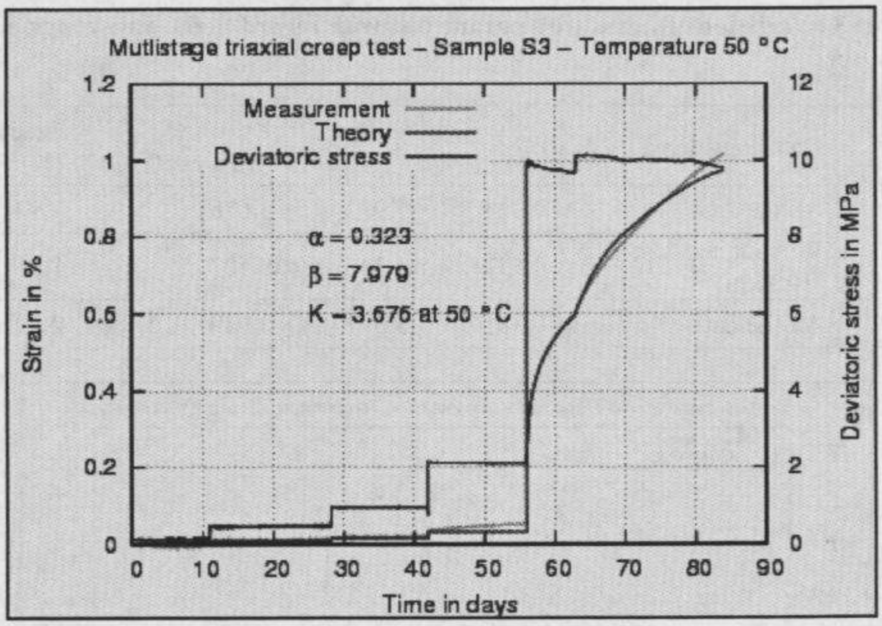

Figure 3 : Experimental and theoretical creep responses of sample S3 (fitting all the data)

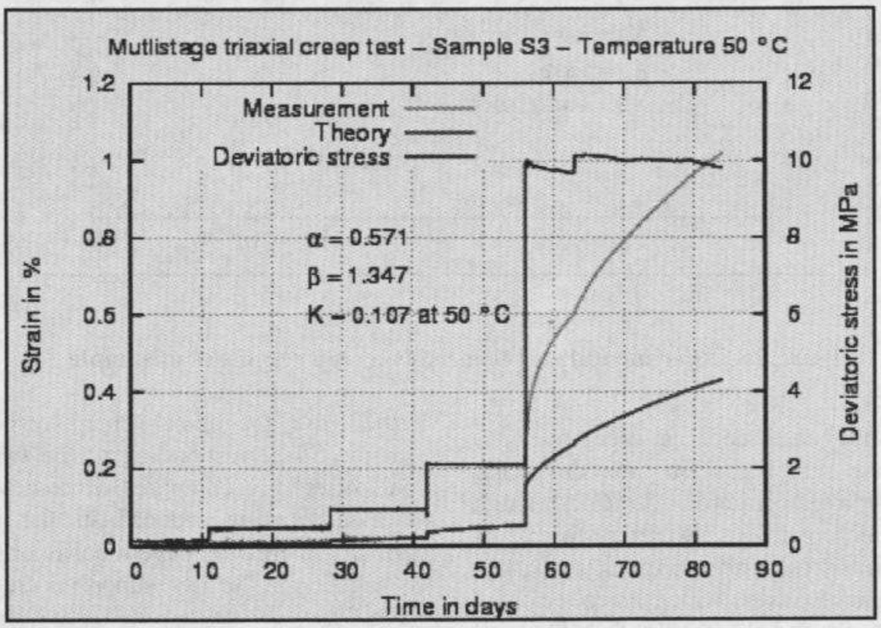

Figure 4 : Experimental and theoretical creep responses of sample S3 (fitting without accounting for the last deviatoric stress stage)

The large variation of Lemaitre's parameters and the corresponding creep indices measured on sample S3, in comparison with the results obtained on the first two samples (S1 and S2) necessitated to check the homogeneity of the salt before proceeding further in the interpretation of the effect of low stresses. An additional creep test was therefore performed on the same salt facies by applying two high steps of the deviatoric stress (5 and 10 MPa) at a confining pressure of 10 MPa. The temperature was also investigated with two steps (50 and 23 °C). The results of this last test are given by table 3 and illustrated by figure 5.

Table 3 : Variation of Lemaitre's parameters with regard to the data considered

Sample	Data considered	α	β	K (MPa at 50° C)	Creep index (%)
S3	All the data	0.323	7.977	3.676	1.97
	Without the last step (10 MPa)	0.571	1.347	0.107	1.31
S4	All the data	0.381	3.392	0.949	2.8

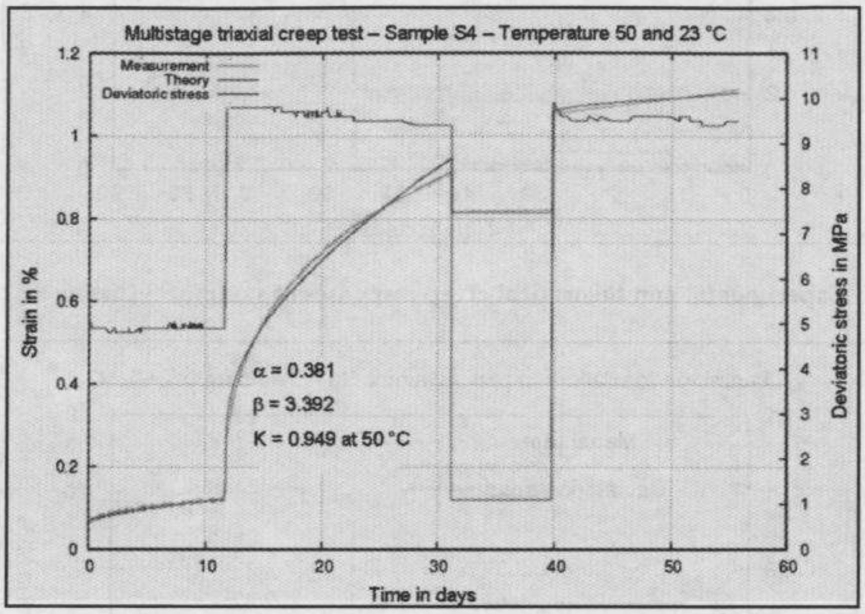

Figure 5 : Experimental and theoretical creep responses of sample S4

Figure 6 gives the theoretical creep response of the four samples versus the deviatoric stress under a temperature of 50 °C. Although the testing conditions were different, it appears clearly that the tested salt is marked by two main facieses, which are constituted by samples S1 and S2, for the first one, and by samples S3 and S4, for the second one respectively.

When the theoretical creep responses of sample S3 corresponding to the two sets of parameters are compared, it comes out that the deformation induced in the range of primary interest for cavern behavior (0-5 MPa) is higher for the low supposed creeping set (Fig. 7).

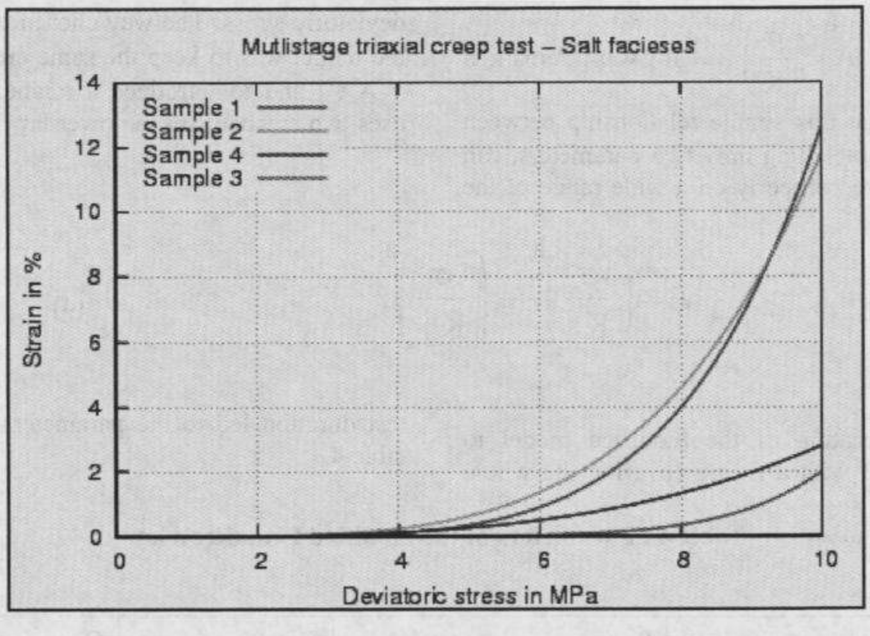

Figure 6 : Comparison of the theoretical creep response of the four tested samples and identification of the salt facieses

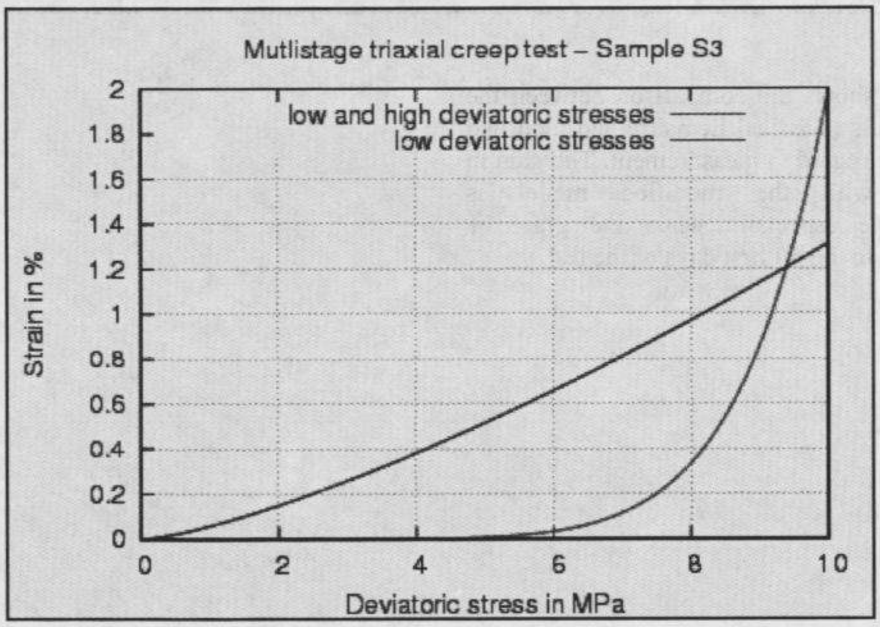

Figure 7 : Comparison of the theoretical creep response of the same salt when tested under high and low deviatoric stresses

4. Improvement of Lemaitre's law

Over the past thirty years, experience showed clearly that when a sample of salt is subjected to a constant deviatoric stress σ and a constant temperature T, the creep strain ε is well represented by a time power law $\varepsilon = A\,t^{\alpha}$, where the exponent α is practically independent of σ and T. Only the factor A is a function of the deviatoric stress and the temperature. The law proposed by Lemaitre (1970) to describe the behavior of ductile metals presents also a creep in the form of t^{α}. This model was applied by Tijani and Vouille (1981) for the creep of the salt rock.

Regarding the influence of the deviatoric stress σ, the Lemaitre's law is defined in a such a way that the factor A is a power function $A = \left(\frac{\sigma}{K}\right)^{\beta}$. It was remarked rapidly that this simple relationship between A and σ, including only two parameters, can not be fitted properly on a wide range of the deviatoric stress. The way chosen to improve the model was to keep the same creep law ($\varepsilon = A\, t^{\alpha}$) and to introduce a relationship that uses two mechanisms in power law :

$$A = \left(\frac{\sigma}{K_1}\right)^{\beta_1} + \left(\frac{\sigma}{K_2}\right)^{\beta_2} \qquad (4)$$

The application of the modified model to sample S3 with a fitting covering the whole test duration led to the parameters given by table 4.

Table 4 : Parameters of the modified Lemaitre's law

Sample	α	β_1	K_1 (MPa at 50° C)	β_2	K_2 (MPa at 50° C)	Creep index (%)
S3	0.323	1.347	0.07	5.826	2.59	2.30

Figure 8 shows the comparison between the two models (classical Lemaitre and modified one) with regard to measurement. The gain in quality with the modified model is remarkable especially when the graph is restricted to the 50 first days of the test.

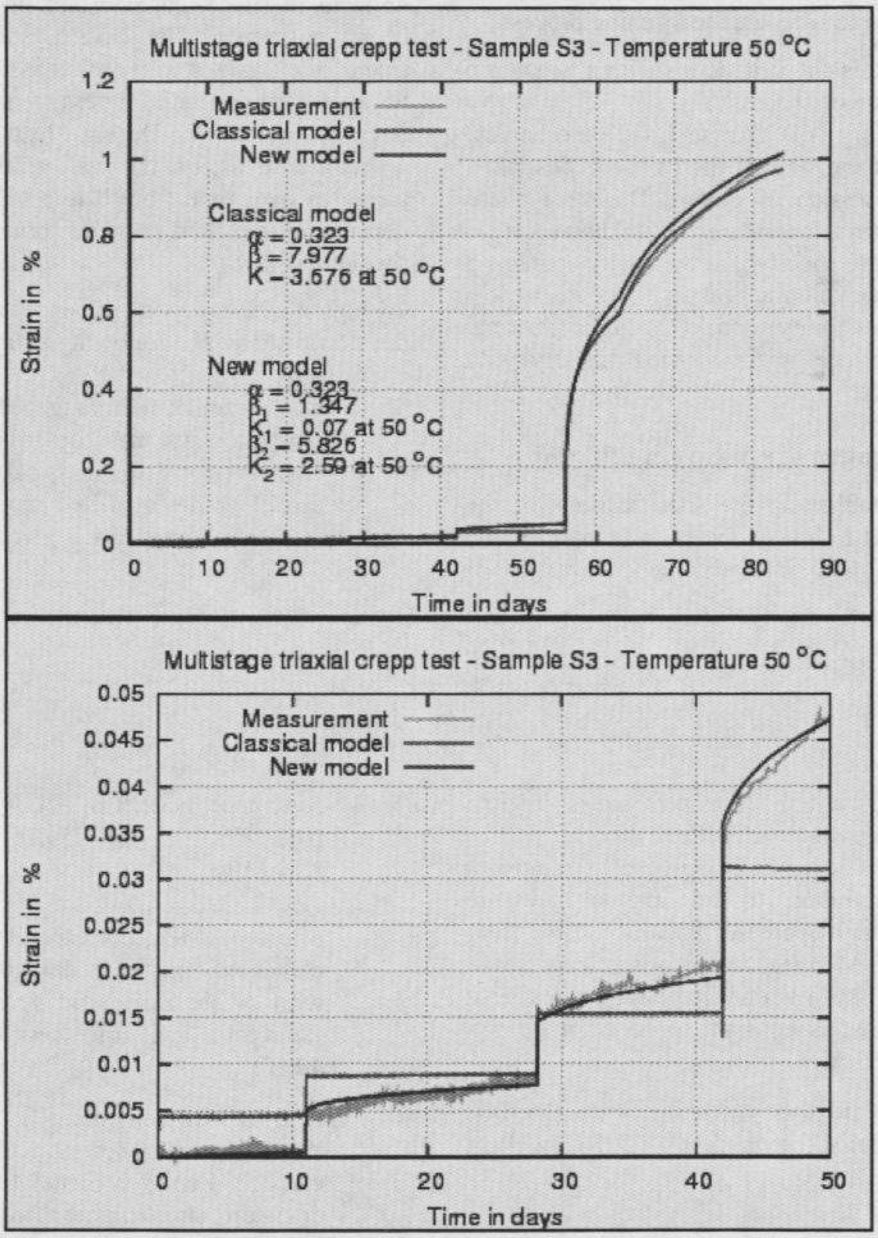

Figure 8 : Comparison of the two models with respect to measurement (the bottom figure shows the behavior within a time period of 50 days)

5. Application of the new model to simulate cavern behavior

5.1 Presentation of the numerical code GEO1D

GEO1D is a software dedicated to solve thermo-mechanical problems of cylindrical and spherical caverns (Tijani 2008). The rock mass surrounding the cavern is homogenous and consists of an isotropic material which is, firstly, a conductor characterized by a thermal conductivity and a volumic heat, and secondly, a deformable material defined by a Young modulus, a Poisson ratio, a linear thermal expansion coefficient and rheological parameters corresponding to an elastic, plastic or viscoplastic behavior (including the modified Lemaitre's law).

The rock mass is initially in a state of uniform temperature and stresses. The cavity is a sphere or a cylinder of a given height which remains constant in time. In both cases, the radius may vary with time between 0 and the final value to simulate the leaching process.

The space is cut in intervals with a geometric progression on the radius.
The history of thermal and mechanical actions at the wall of the cavity is described by a succession of phases. During a phase where the temperature is unknown, it is necessary to specify that the cavity contains a fluid with a volumic heat,
a compressibility and a volumic thermal expansion coefficient varying linearly with temperature.

5.2 Simulation of salt cavern behavior

When establishing the specifications of the creep tests to be conducted in laboratory, the engineers take into account the ranges of loading (temperature and deviatoric stress) which are representative of the conditions surrounding the caverns to be achieved.

For this demonstration, a temperature, uniform in space and constant in time, has been considered. The cavern, spherical in shape, is excavated in a rock mass submitted to an isotropic initial stress regime (zero deviatoric stress). During leaching, the deviatoric stress, on the wall of the cavity, increases over time from 0 to a value which depends on the fluid pressure (brine) and the rheological law.

To highlight the strong influence of the low deviatoric stress, the following simple case is considered :

- an infinite, homogeneous and isotropic rock mass described by the elasto-viscoplastic classical Lemaitre model or the modified model whose isothermal creep law is given by :

$$\varepsilon = \left[\left(\frac{\sigma}{K_1} \right)^{\beta_1} + \left(\frac{\sigma}{K_2} \right)^{\beta_2} \right] t^{\alpha} \qquad (5)$$

- an initial isotropic stress regime (pressure = 25 MPa)
- a spherical cavern (radius=a) submitted to an internal pressure which decreases linearly in time from 25 MPa (the initial isotropic stress) to 10 MPa within 365 days, thereafter it remains constant at 10 MPa for t > 365 days.

Regarding the rheological behavior, 3 models are considered for which $\alpha = 0.323$ (Fig. 9).

- Reference model : $\beta_1 = 1.347$ - $K_1 =$ 0.07 MPa - $\beta_2 = 5.826$ - $K_2 =$ 2.59 MPa
- Low-Lemaitre model : $\beta = 1.347$ - K = 0.07 MPa, corresponding to the classical Lemaitre model with a single term of the deviatoric stress power, fitted for a deviatoric stress lower than 4 MPa.
- High-Lemaitre model : $\beta = 4.695$ - K = 1.768 MPa, corresponding to the classical Lemaitre model fitted for a deviatoric stress higher than 8.5 MPa.

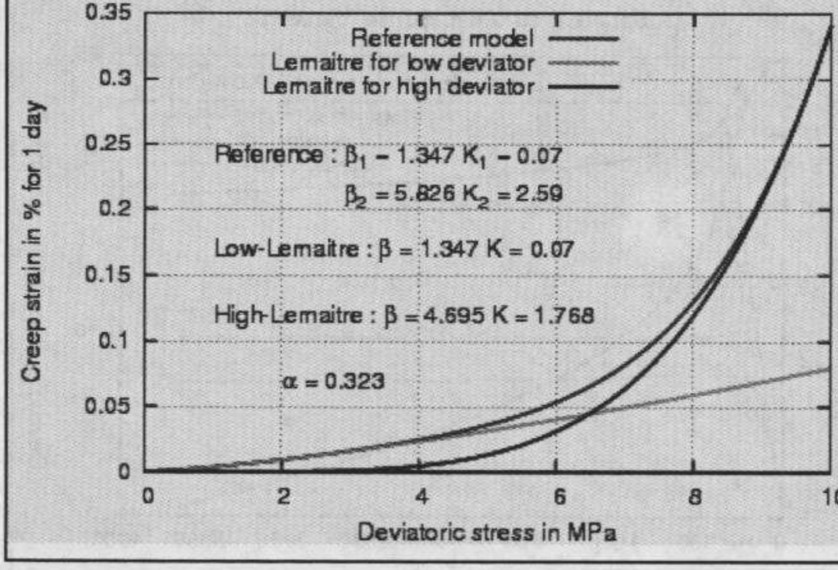

Figure 9 : The three models used for cavern simulation

Figure 10 shows the variation of the deviatoric stress with time at the wall of the cavern for the three considered models, in comparison with the elastic behavior. It appears clearly that in case of viscosity, the deviatoric stresses are lower than the elastic case. This relaxation phenomenon emphasizes again on the importance of accounting for low deviatoric stresses in the creep law.

The representation of the relative variation of volume on the same figure indicates that the classical Lemaitre model gives an answer relatively far from the modified model for this parameter too. Assuming that the reference model gives the "true" response behaviour of the cavern, fitting the cavern volume variation with the classical Lemaitre law will entail a significant overestimation of the salt creep ability. The Lemaitre modified law is therefore recommended for the interpretation of in-situ tests carried out on caverns for large scale salt creep characterization.

The analysis of the spatial distribution (distance to the centre of the cavern scaled by the inner radius) of the deviatoric stress at the time of 3650 days shows clearly that the high deviatoric stresses (> 4 MPa) covers only a limited thickness around the cavern (maximum 2 times the radius, Fig. 11).

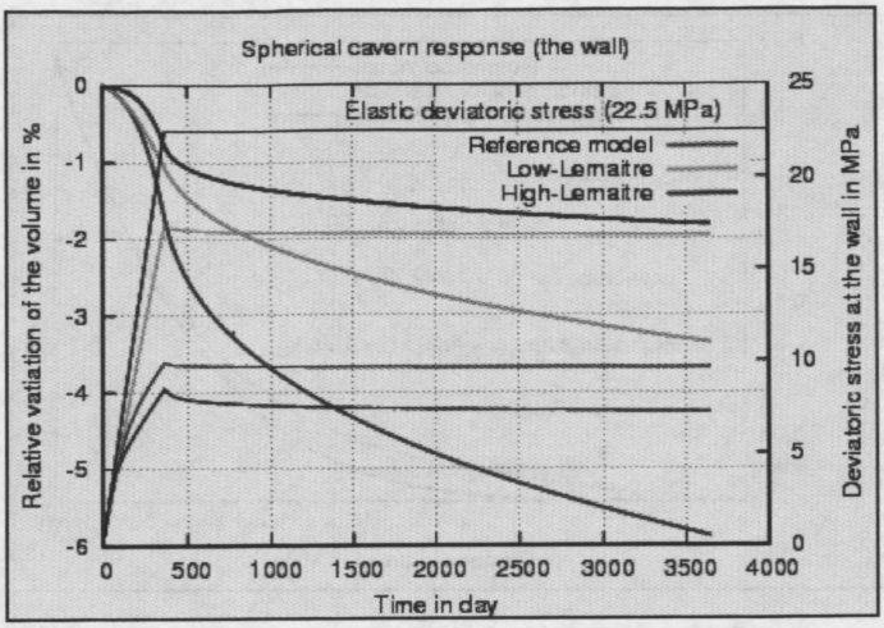

Figure 10 : Comparison of the deviatoric stress at the cavern wall and the volumic variation for the different simulated models

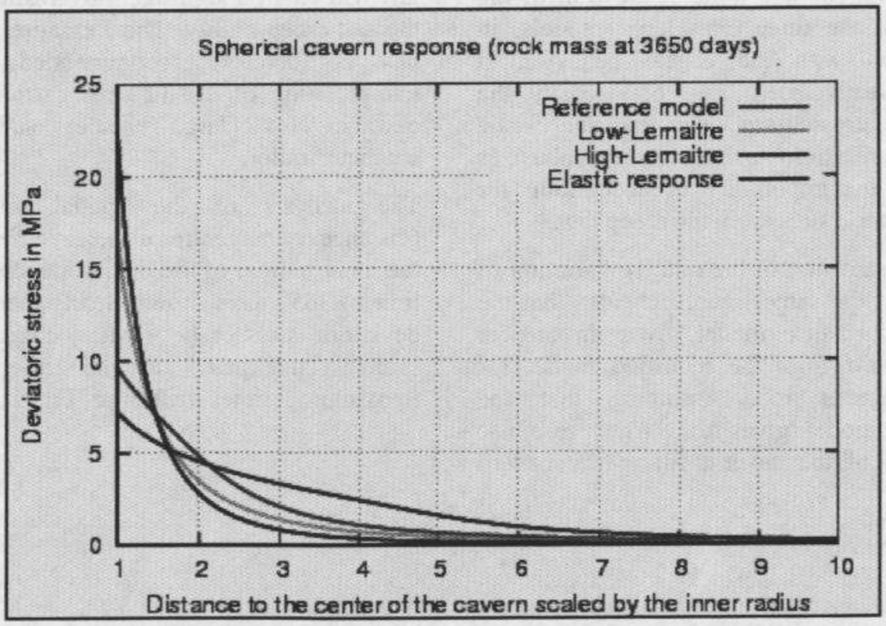

Figure 11 : Comparison of the spatial deviatoric stress distribution at the date of 10 years

6. Conclusions

This paper has clearly demonstrated the importance of low deviatoric stresses in the behavior of salt caverns. The modified Lemaitre law proposed takes accounts for both low and high deviatoric stresses by integrating two mechanisms in power law.

Two main conclusions are noteworthy. The first one concerns the need to perform stages with low deviatoric stresses during the laboratory creep tests. The second conclusion is related to the rheological modeling. When the model to be used does not incorporate a wide range of the deviatoric stress, it is preferable to allocate two sets of parameters obtained by making a global fitting on the whole range and by fitting only the lower values of the deviatoric stresses. The cavern analyses have to be carried out with both sets of parameters and pessimistic conclusions of the two models should be considered.

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